

Coulomb drag in graphene

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We calculate theoretically the Coulomb drag resistivity for two graphene monolayers spatially separated by a distance “ d ”. We show that the frictional drag induced by inter-layer electron-electron interaction goes asymptotically as T^2/n^3 and $T^2 \ln(n)/n$ in the high-density ($k_F d \gg 1$) and low-density ($k_F d \ll 1$) limits, respectively.

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Frictional drag measurements of transresistivity in double layer systems have led to significant advances in our understanding of density and temperature dependence of electron-electron interactions in 2D systems [1]. Recent interest has focused on the role of electron interaction effects on the graphene drag resistivity, which should vary in a systematic manner as a function of electron density (n), layer separation (d), and temperature (T). In particular, a recent experiment of Coulomb drag in double layer graphene by Kim *et al.* [2] is particularly interesting. In view of the considerable fundamental significance of the issues raised by the experimental observations, we present in this paper a careful theoretical calculation of frictional drag $\rho_D(T)$ in a 2D [3] graphene within the canonical many-body Fermi liquid theory. The current work is a generalization of the earlier theoretical work on graphene drag by Tse *et al.* [4].

We start by writing down the theoretical formula for ρ_D [4] in the many-body Fermi liquid RPA-Boltzmann theory approximation widely used in the literature. The double layer frictional drag in a many body-Fermi liquid diagrammatic perturbation theory with dynamically

screened electron-electron interaction is given by [4]

$$\rho_D = \frac{\hbar^2}{2\pi e^2 n^2 k_B T} \int \frac{q^2 d^2 q}{(2\pi)^2} \int d\omega \frac{F_1(q, \omega) F_2(q, \omega)}{\sinh^2(\beta\omega/2)}, \quad (1)$$

where $F_{1,2}(q, \omega) = |u_{12}^{sc}(q, \omega)| \text{Im} \Gamma_{11,22}(q, \omega)$, with $u_{12}^{sc} = v_{12}^c / \epsilon(q, \omega)$ is the dynamically screened interlayer Coulomb interaction between layers, and $\Gamma(q, \omega)$ is the 2D graphene non-linear susceptibility [4]. The interlayer Coulomb interaction is given by $v_{12}^c(q) = v^c(q) e^{-qd}$ with the intralayer Coulomb potential [3] $v_c(q) = 2\pi e^2 / \kappa q$ where κ is the background dielectric constant. (We consider the so-called balanced situation here with the same carrier density n in both layers.) Note that the dielectric function $\epsilon(q, \omega)$ entering Eq. (1) is given by [3]

$$|\epsilon(q, \omega)| = [1 - v_{11}(q) \Pi_{11}(q, \omega)] [1 - v_{22}(q) \Pi_{22}(q, \omega)] - v_{12}(q) v_{21}(q) \Pi_{11}(q, \omega) \Pi_{22}(q, \omega), \quad (2)$$

where $v_{ii}(q) = v_c(q)$ and Π_{ii} is the intralayer graphene polarizability [5]. Even though there are analytic expressions for graphene polarizability at $T = 0$, [5] the finite temperature versions of the polarizability have not been calculated analytically. The full expression of finite temperature polarizability is necessary to understand more precisely the temperature dependent drag including the plasmon enhancement effects [6]. Here we provide the efficient way for calculating the finite temperature graphene polarizability by generalizing our earlier work [5]

$$\Pi(q, w, T) = \frac{\pi}{8} \frac{q^2}{\sqrt{q^2 - \omega^2}} + \int_0^\infty dk [f(q) + g(q)] \left[\frac{\left(\frac{\omega^2 - q^2}{4} + \omega k + k^2 \right) \text{sgn}(a_+)}{\sqrt{\frac{(\omega^2 - q^2)^2}{4} + (\omega^2 - q^2)(\omega k + k^2)}} + \frac{\left(\frac{\omega^2 - q^2}{4} - \omega k + k^2 \right) \text{sgn}(a_-)}{\sqrt{\frac{(\omega^2 - q^2)^2}{4} + (\omega^2 - q^2)(-\omega k + k^2)}} \right], \quad (3)$$

where $\Pi = \Pi/D_0$ ($D_0 = 2k_F/\pi\hbar v_F^2$ is the density of states of graphene at Fermi energy, and k_F and v_F are the Fermi wave vector and Fermi velocity of graphene), $q = q/k_F$, $\omega = \omega/E_F$, $f(q) = [e^{-(\epsilon_q - \mu)} - 1]^{-1}$, $g(q) = [e^{(\epsilon_q + \mu)} + 1]^{-1}$, $\epsilon_q = \hbar v_F q$, and $a_\pm = \omega^2 - q^2 \pm 2\omega k$. In $f(q)$ and $g(q)$, μ is the finite temperature chemical

potential which must be calculated self-consistently to conserve the total electron density.

With assumptions of a large inter-layer separation ($k_F d \gg 1$, or $q_{TF} d \gg 1$, with q_{TF} being the Thomas Fermi (TF) screening wave vector) and the random phase approximation (RPA) in which Π_{ii} is replaced by its value

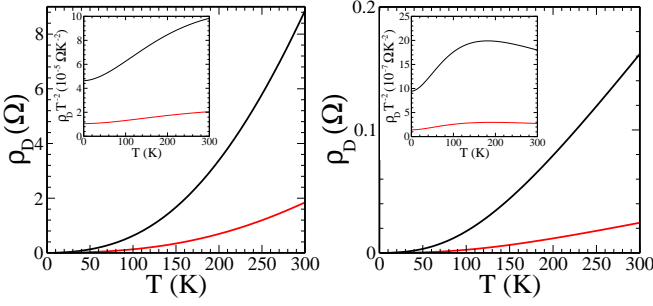


FIG. 1. The temperature dependence of Coulomb drag for (a) $d = 50 \text{ \AA}$ and (b) $d = 200 \text{ \AA}$. The black (red) lines are the results calculated with equal electron densities, $n_1 = n_2 = 10^{12} \text{ cm}^{-2}$ ($n_1 = n_2 = 2 \times 10^{12} \text{ cm}^{-2}$). Insets show the scaled drag resistivity $\rho_D(T)/T^2$.

for the non-interacting electrons, we have for the identical layers at high density and low temperature,

$$\rho_D = \frac{\hbar}{e^2} \frac{\zeta(3)\pi}{16} \frac{(T/T_F)^2}{(k_F d)^2 (q_{TF} d)^2} \quad (4)$$

where $q_{TF} = 4r_s k_F$ is the TF wave vector with the graphene fine structure constant $r_s = e^2/\kappa\hbar v_F$ and ζ is the Riemann zeta function. This result shows that $\rho_D(n) \propto n^{-3}$, but this result, which was obtained in ref. [4], applies only for high density and large separation limits (or weak interlayer correlation, $k_F d \gg 1$). At low densities (or strong interlayer correlation, $k_F d \ll 1$) the exponent in the density dependent drag differs from -3 as shown in Fig. 2 where we directly numerically calculated graphene drag using Eqs. (1)-(3). Eq. 4 shows $\rho_D(T) \propto T^2$. For large layer separation (i.e. $k_F d \gg 1$) the back-scattering $q \approx 2k_F$ is suppressed due to the exponential dependence of the interlayer Coulomb interaction $v_{12}(q) \propto \exp(-qd)/q$ as well as the graphene chiral property. In this case the drag is dominated by small angle scattering and one expects $\rho_D \propto T^2/(n^3 d^4)$.

For the strong interlayer correlation ($k_F d \ll 1$) in the low-density or small-separation limit, we have the following asymptotic behavior:

$$\rho_D = \frac{\hbar}{e^2} \frac{8\pi r_s^2}{3} \frac{T^2}{T_F^2} \ln \left[\frac{(2q_{TF} d + 1)^2}{4q_{TF} d(1 + q_{TF} d)} \right]. \quad (5)$$

Thus, we have $\rho_D(T) \sim T^2$ again, and $\rho_D(n) \sim \ln(n)/n$ with very weak logarithmic d -dependence.

In Fig. 1 we show the calculated Coulomb drag as a function of temperature for two different densities $n = 1, 2 \times 10^{12} \text{ cm}^{-2}$, layer separation (a) $d = 50 \text{ \AA}$ (b) $d = 200 \text{ \AA}$. The overall temperature dependence of drag is close to the quadratic behavior. But we find a small corrections, especially at low values of $k_F d$. In regular 2D systems there is a $\ln(T)$ corrections to the T^2 dependence of the drag. However, due to the suppression of the back-scattering in graphene such logarithmic correction does

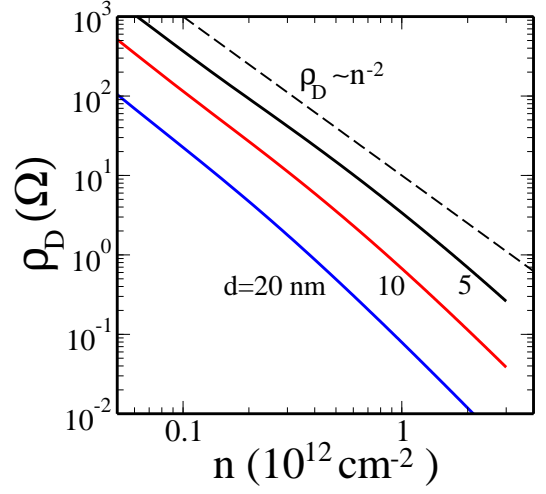


FIG. 2. The density dependent Coulomb drag for different layer separations $d = 5, 10, 20 \text{ nm}$ and $T = 200 \text{ K}$. Dashed line indicates $\rho_D \sim n^{-2}$ behavior.

not show up in our numerical results except perhaps at extremely low temperatures.

In Fig. 2 the density dependent Coulomb drag is shown for different layer separations. Our calculated Coulomb drag resistivity follow a n^α dependence with $\alpha \lesssim -2$ at low carrier densities (or, $k_F d < 1$), but as the density increases the exponent (α) decrease. Based on our calculation we believe that the experimental departure from the n^{-3} behavior reported in Ref. [2] is essentially a manifestation of the fact that the asymptotic n^{-3} regime is hard to reach in low density electron systems where $k_F d \gg 1$ limit simply cannot be accessed. We predict a weak $\ln(n)/n$ density dependence in the low-density or small separation limit.

In conclusion, we study the frictional drag between two spatially separated graphene layers within a many-body Fermi liquid theory. We find that the temperature dependent drag mostly shows a quadratic behavior regardless of the layer separation, but the density dependence varies from $\ln(n)/n$ for $k_F d \ll 1$ to n^{-3} for $k_F d \gg 1$. But most currently available double layer graphene samples belong to $k_F d \sim 1$, so the density dependence does not have any universal power law behavior. We also find that due to the suppression of the $q = 2k_F$ back-scattering there is no $\ln(T)$ correction in the drag resistivity in graphene.

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